Chapter 2

Complex Numbers

Chapter Outline

2.1 INTRODUCTION AND DEFINITIONS

A collection of real number system was not complete. We realize that a quadratic equation $a^2x + bx + c = 0$, could not be solved if $b^2 - 4ac < 0$. Consider a quadratic equation $x^2 + 1 = 0, a = c = 1, b = 0$ then we have $x = +\sqrt{-1}$ and $x = -\sqrt{-1}$, Since we know that the square of any real number must be greater than or equal than zero, we realize that the real number system was not complete. This leads to a study of complex numbers which are useful in variety of applications. A complex number system allowed the possibility of negatives squares and had the real numbers as a subset, *R*. In above quadratic equation we do not have any problems in solving this equation as complex number system allowed a negative square. We introduce a symbol, j , with the property that $j^2 = -1$ and it follows $j = \sqrt{-1}$. We can use this to write down the square root of any negative numbers. Then the solution of the equation are given by $x = j$ and $x = -j$.

Example: Write down the expression of the $\sqrt{9}$ and -9.

Solution:

$\sqrt{9} = \pm 3$

$\sqrt{-9} = \sqrt{9 \times -1} = \sqrt{9} \times \sqrt{-1} = \pm 3 \times \sqrt{-1} = \pm 3j$

- Simplify j^3, j^4, j^8

Definition 2.1: Complex Number

If z is a complex number then we write

$$z = a + bj$$

where $a, b \in R$, a is a real part and b is the imaginary part.

Example 1

Express in the form $z = a + bj$.

a) $\sqrt{-25}$ b) $\sqrt{-20}$

c) $5 + \sqrt{-36}$ d) $\sqrt{27} + \sqrt{-18}$

Solution:

a) $\sqrt{-25} = \sqrt{25(-1)} = \sqrt{25}\sqrt{-1} = 5j$

b) $\sqrt{-20} = \sqrt{4(5)(-1)} = 2\sqrt{5}j$

c) $5 + \sqrt{-36} = 5 + 6j$

d) $\sqrt{27} + \sqrt{-18} = 3\sqrt{3} + \left(3\sqrt{2}\right)j$

Definition 2.2: Equal Complex Numbers

Two complex numbers $z_1 = a + bj$ and $z_2 = c + dj$ are equal if and only if $a = c$ and b = d.

Example 2

Given $2x + 3yj = 4 + 3j$. Find x and y.

Solution:

$x = 2$ and $y = 1$

Definition 2.3: Operations of complex numbers

If $z_1 = a + bj$ and $z_2 = c + dj$ then,

i) $z_1 + z_2 = (a + c) + (b + d)j$

ii) $z_1 - z_2 = (a - c) + (b - d)j$

iii) $z_1 \times z_2 = (a + bj)(c + dj)$

$= ac + adj + bcj + bdj^2$

$= (ac - bd) + (ad + bc)j$

Example 3

Given $z_1 = 2 + 3j$ and $z_2 = 1 - j$. Find

a) $z_1 + z_2$

b) $z_1 - z_2$

c) $z_1 * z_2$

d) Determine the value of $z = 7(3 + 4j) - (2 + j)(5 - 6j)$

Solution:

a) $z_1 + z_2 = (2 + 3j) + (1 - j) = (2 + 1) + (3 - 1)j = 3 + 2j$

b) $z_1 - z_2 = (2 + 3j) - (1 - j) = (2 - 1) + (3 + 1)j = 1 + 4j$

c) $z_1 * z_2 = (2+3j)(1-j) = 2 - 2j + 3j - 3j^2 = 5 + j$

d) $z = 7(3 + 4j) - (2 + j)(5 - 6j)$

$$
\begin{aligned}
&= (21 + 28j) - (10 + 5j - 1j - 6j^2) \\
&= (21 + 28j) - (10 + 6 + (5 - 12j)) \\
&= (21 + 28j) - (16 + (-7j)) \\
&= (21 - 16) + (28 + 7)j \\
&= 5 + 35j
\end{aligned}
$$

Definition 2.4: Complex Conjugate

If $z = a + bj$, its complex conjugate, denoted by $\bar{z}$, is

$$\bar{z} = a - bj$$

* Show that $z\bar{z} = a^2 + b^2$

Example 4

Find the complex conjugate of:

a) $z = 4 + 5j$ b) $z = 4 - 5j$

c) $z = -j$ d) $z = j$

Solution:

a) $z = 4 - 5j$ b) $z = 4 + 5j$

c) $z = j$ d) $z = -j$

Properties of conjugate complex number:

a) $z = \overline{\overline{z}}$ b) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

c) $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$ d) $\overline{z_1 \times z_2} = \overline{z_1} \times \overline{z_2}$

e) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{\overline{z_2}}}$ f) $\frac{1}{\overline{\overline{z}}} = \overline{\left(\frac{1}{z}\right)}$ g) $\overline{z^n} = \left(\overline{z}\right)^n$: n integer

h) $\frac{z + \overline{z}}{2} = \text{Re}(z)$ i) $\frac{z - \overline{z}}{2} = \text{Im}(z)$

Definition 2.5: Division of complex numbers

If $z_1 = a + b$ and $z_2 = c + d$ then

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{a + bj}{c + dj} \\ &= \frac{a + bj}{c + dj} \times \frac{c - dj}{c - dj} \\ &= \frac{ac + bd + (bc - ad)j}{c^2 + d^2} \end{aligned}$$

Example 5:

Determine the value of:

a) $\frac{2 - j}{1 + j}$

b) $\frac{1 - j}{3 + j}$

c) $z = \frac{7}{3 - j} - \frac{1 - j2}{3 + 4j}$

Solution:

a) $\dfrac{2-j}{1+j}=\left(\dfrac{2-j}{1+j}\right)\left(\dfrac{1-j}{1-j}\right)=\dfrac{2-2j-j+j^2}{1-j^2}=\dfrac{1}{2}-\dfrac{3}{2}j$

b) $\dfrac{1-j}{3+j}=\left(\dfrac{1-j}{3+j}\right)\left(\dfrac{3-j}{3-j}\right)=\dfrac{3-j-3j+j^2}{9-j^2}=\dfrac{1}{5}-\dfrac{2}{5}j$

c) $z=\dfrac{7}{3-j}-\dfrac{1-j2}{3+4j}$

$$
\begin{aligned}
&=\frac{7(3+j)}{(3-j)(3+j)}-\frac{(1-j2)(3-4j)}{(3+4j)(3-4j)}\\
&=\frac{(21+7j)}{(9+j3-3j-j^2)}-\frac{(3-j6-4j+8j^2)}{(9+12j-j12-16j^2)}\\
&=\frac{(21+7j)}{(9+1)}-\frac{(3-8-j10)}{(9+16)}\\
&=(2.1+j(0.7))+(0.2+j(0.4))\\
&=2.3+1.1j
\end{aligned}
$$

2.2 ARGAND DIAGRAM

The complex number $z=a+bj$ is plotted as a point with coordinates (a,b). Such a diagram is called an Argand diagram.

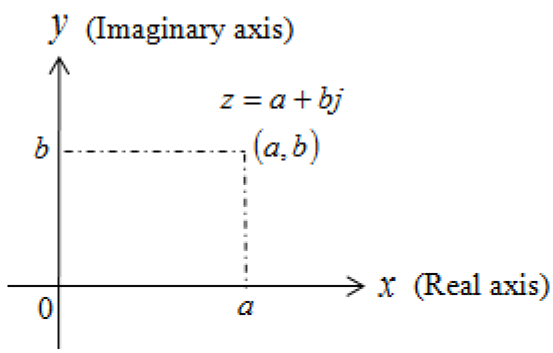


Figure 1

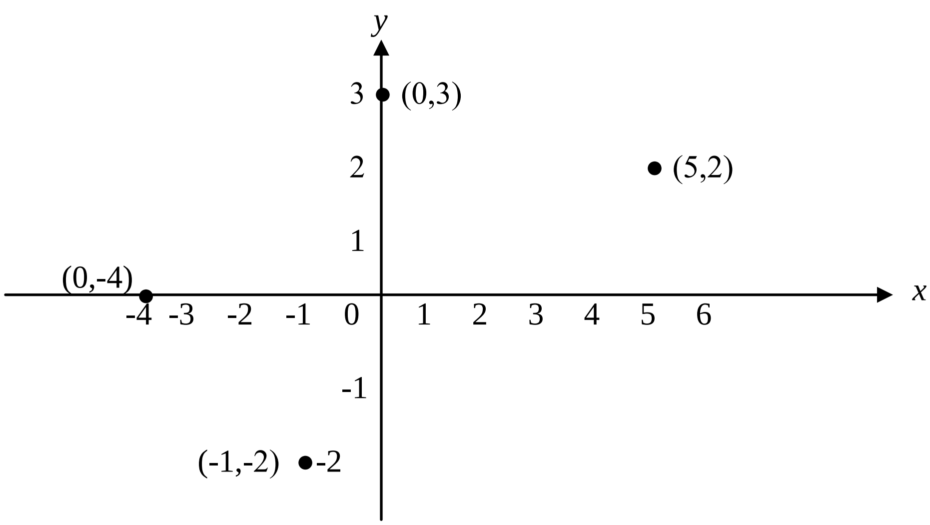
Example 6

Plot the following complex numbers on an Argand diagram.

a) $3j$ b) $5 + 2j$

c) -4 d) $-1 - 2j$

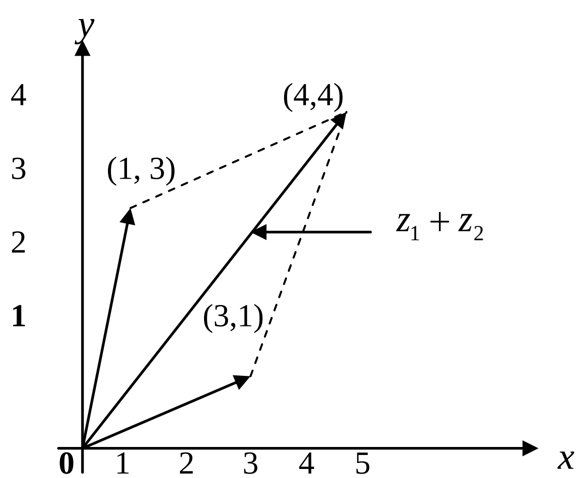
Solution:



Example 7

Given that $z_1 = 1 + 3j$ and $z_2 = 3 + j$. Plot $z_1 + z_2$ in an Argand diagram.

Solution:



2.3 THE MODULUS AND ARGUMENT OF A COMPLEX NUMBERS

Consider an Argand diagram in Figure 2, the distance of the point (a,b) imaginary axis and real axis.

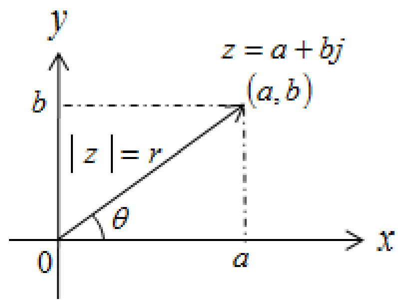


Figure 2

> **Definition 2.6: Modulus of complex numbers**
> The modulus of a complex number $z = a + bj$ written as $|z|$ is
>
> $$r = |z| = \sqrt{a^2 + b^2}$$

Example 8

Find the modulus of the following complex numbers.

a) $3 - 4j$ b) $5 + 2j$

c) $-1 - j$ d) $7j$

Solution:

a) $|3 - 4j| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

b) $|5 + 2j| = \sqrt{5^2 + 2^2} = \sqrt{29}$

c) $|-1 - j| = \sqrt{2}$

d) $|7j| = 7$

Properties of Modulus

a) $|\bar{z}| = |z|$

b) $z\bar{z} = |z|^2$

c) $|z_1 z_2| = |z_1||z_2|$

d) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}, z_2 \neq 0.$

e) $|z^n| = |z|^n$

f) $|z_1 + z_2| \leq |z_1| + |z_2|$

g) $|z_1 + z_2| \geq |z_1| - |z_2|$

Definition 2.7: Arguments of complex numbers

The argument of the complex number, $z = a + bj$ often given the symbol θ is define as

$$\theta = \tan^{-1}\left(\frac{b}{a}\right).$$

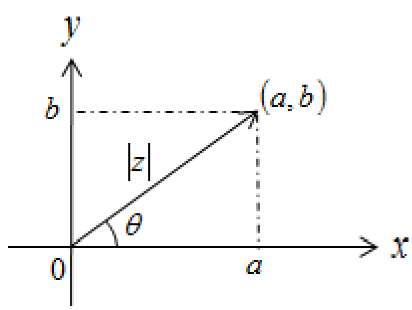


Figure 3

Example 9

Find the arguments of the following complex numbers;

a) $3 - 4j$

b) $5 + 2j$

c) $-1 - j$

d) $1 + 7j$

Solution:

If θ is the argument of the following complex numbers, therefore,

a) $\theta = \tan^{-1}\left(\dfrac{-4}{3}\right) = -53.13^{o} = 306.87^{o}$

b) $\theta = \tan^{-1}\left(\frac{2}{5}\right) = 21.80^{o}$

c) $\theta = \tan^{-1}\left(\frac{-1}{-1}\right) = 225^{o}$

d) $\theta = \tan^{-1}\left(\frac{7}{1}\right) = 81.87^{o}$

2.4 THE POLAR FORM OF COMPLEX NUMBERS

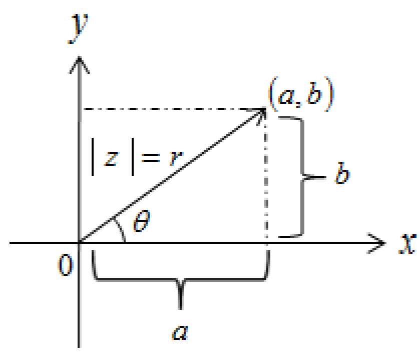


Figure 4

Using trigonometry we can write,

$$\sin\theta = \frac{b}{r} \; ; \; b = r\sin\theta$$

$$\cos\theta = \frac{a}{r} \; ; \; a = r\cos\theta$$

$$\tan\theta = \frac{b}{a} \; ; \; \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

and; $|z| = r = \sqrt{a^2 + b^2}$

Therefore, we can then write $z = a + bj$ in polar form as:

$$z = r\cos\theta + j\,r\sin\theta$$

$$= r(\cos\theta + j\sin\theta) \qquad \text{or} \qquad = (r, \theta)$$

Example 10

State the complex numbers of z when

a) $z = 1 + j$ b) $z = 3 - 4j$

Solution:

a) $r = \sqrt{1^2 + 1^2} = \sqrt{2}$ and $\theta = \tan^{-1}(1) = 45^o$

Hence, $z = \sqrt{2}\,(\cos 45^o + j \sin 45^o)$ or $z = (\sqrt{2}, 45^o)$

b) $r = \sqrt{3^2 + (-4)^2} = 5$ and $\theta = \tan^{-1}\left(\frac{-4}{3}\right) = -53.13^o$ @ 306.87^o

Hence, $z = 5(\cos 306.87^o + j \sin 306.87^o)$ or $z = (5, 306.87^o)$

Exercise 11

State the following complex numbers into the form of $z = a + bj$.

a) $z = 4(\cos 30^o + j \sin 30^o)$ b) $z = \sqrt{3}\,(\cos 135^o + j \sin 135^o)$

Solution:

a) $r = 4$ dan $\theta = 30^o$

$a = r\cos\theta = 4\cos 30^o = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$b = r\sin\theta = 4\sin 30^o = 4\left(\frac{1}{2}\right) = 2$

Hence, $z = 4(\cos 30^o + j \sin 30^o) = 2\sqrt{3} + 2j$

b) $r = \sqrt{3}$ dan $\theta = 135^o$

$a = r\cos\theta = \sqrt{3}\cos 135^o = \sqrt{3}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\sqrt{3}}{\sqrt{2}}$

$b = r\sin\theta = \sqrt{3}\sin 135^o = \sqrt{3}\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{\sqrt{2}}$

Hence, $z = \sqrt{3}\,(\cos 135^o + j \sin 135^o) = -\frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}}j$

Definition 2.8:

If z_1 and z_2 are two complex numbers in polar form where $z_1 = r_1(\cos\theta_1 + j\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + j\sin\theta_2)$ therefore

i) $z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2) + j\sin(\theta_1+\theta_2)]$

ii) $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1-\theta_2) + j\sin(\theta_1-\theta_2)]$

Example 12

Given that $z_1 = 4(\cos 30^o + j \sin 30^o)$ and $z_2 = 2(\cos 60^o + j\sin 60^o)$. Find

a) $z_1 z_2$ b) $\dfrac{z_1}{z_2}$

Solution:

a) $z_1 z_2 = 4.2\,[\cos(30^o + 60^o) + j\sin(30^o + 60^o)]$

$= 8(\cos 90^o + j\sin 90^o)$

$= 8j$

b) $\dfrac{z_1}{z_2} = \dfrac{4}{2}[\cos(30^o - 60^o) + j\sin(30^o - 60^o)]$

$= 2[\cos(-30^o) + j\sin(-30^o)]$

$= 2\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}j\right)$

$= \sqrt{3} - j$

2.5 THE EXPONENTIAL FORM OF COMPLEX NUMBERS

Definition 2.9:

Complex number $z = r(\cos\theta + j\sin\theta)$ can also be written in exponential or Euler form as $z = re^{\theta j}$ where θ is measured in radians and $e^{\theta j} = \cos\theta + j\sin\theta$.

Example 13

State the following angles θ in radians.

a) 45^o b) 120^o c) 270^o

Solution:

a) $45^0 = 45^0 . \dfrac{\pi}{180^0} = \dfrac{\pi}{4}$

b) $120^0 = 120^0 . \dfrac{\pi}{180^0} = \dfrac{2\pi}{3}$

c) $270^0 = 270^0 . \dfrac{\pi}{180^0} = \dfrac{3\pi}{2}$

Example 14

Express the complex number $z = 1 + j$ in exponential form.

Solution:

From Example 10 (a), we know that $z = 1 + j = \sqrt{2}\,(\cos 45^o + j \sin 45^o)$ which

$r = \sqrt{2}$ dan $\theta = 45^o$.

Then we have $45^0 = \dfrac{\pi}{4}$.

Hence, $z = 1 + j = \sqrt{2}\,(\cos 45^o + j \sin 45^o)$

$$= \sqrt{2} e^{\frac{\pi}{4} j}$$

Definition 2.10:

If $z_1 = r_1 e^{\theta_1 j}$ and $z_2 = r_2 e^{\theta_2 j}$, then

i) $z_1 z_2 = r_1 r_2 e^{(\theta_1 + \theta_2) j}$

ii) $\dfrac{z_1}{z_2} = \dfrac{r_1 e^{(\theta_1 - \theta_2) j}}{r_2}, \; z_2 \neq 0$

Example 15

If $z_1 = 8e^{\frac{\pi}{2}j}$ and $z_2 = 2e^{\frac{\pi}{3}j}$, find

a) $z_1 z_2$ b) $\dfrac{z_1}{z_2}$

Solution:

a) $z_1 z_2 = 8.2e^{\left(\frac{\pi}{2}+\frac{\pi}{3}\right)j} = 16e^{\frac{5\pi}{6}j}$

b) $\dfrac{z_1}{z_2} = \dfrac{8e^{\left(\frac{\pi}{2}-\frac{\pi}{3}\right)j}}{2} = 4e^{\frac{\pi}{6}j}$

2.6 DE MOIVRE'S THEOREM

Definition 2.11:

If $z = r(\cos\theta + j\sin\theta)$ is a complex number in polar form to any power n, then

$$z^n = r^n(\cos n\theta + j\sin n\theta)$$

with any value n.

Example 16

If $z = \cos 30^\circ + j\sin 30^\circ$, find z^3 and z^5.

Solution:

a) $z^3 = (\cos 30^\circ + j\sin 30^\circ)^3$

$= \cos 3(30^\circ) + j\sin 3(30^\circ)$

$= \cos 90^\circ + j\sin 90^\circ$

$= j$

b) $z^5 = (\cos 30^\circ + j\sin 30^\circ)^5$

$= \cos 5(30^\circ) + j\sin 5(30^\circ)$

$= \cos 150^\circ + j\sin 150^\circ$

$= -\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}j$

Example 17

If $z = 2(\cos 25^o + j \sin 25^o)$. Calculate

a) z^5 b) z^{-3} c) $z^{\frac{2}{3}}$

Solution:

a) $z^5 = [2(\cos 25^o + j \sin 25^o)]^5$

$= 2^5[\cos 5(25^o) + j \sin 5(25^o)]$

$= 32(\cos 125^o + j \sin 125^o)$

$= 32(-0.5736 + 0.8192j)$

$= -18.355 + 26.214j$

b) $z^{-3} = 2^{-3}(\cos -75^o + j \sin -75^o)$

$= \frac{1}{8}(0.2588 – 0.9659j)$

$= 0.0324 – 0.1207j$

c) $z^{\frac{2}{3}} = 2^{\frac{2}{3}}[\cos \frac{2}{3}(25^o) + j \sin \frac{2}{3}(25^o)]$

$= 1.587(\cos 16.67^o + j \sin 16.67^o)$

$= 1.52 + 0.455j$

2.7 FINDING ROOTS OF COMPLEX NUMBERS

Definition 2.12:
If $z = r(\cos \theta + j \sin \theta)$ then, the n root of z is

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}(kos\frac{\theta + 360^o k}{n} + j\sin\frac{\theta + 360^o k}{n}) \text{ if } \theta \text{ in degrees}$$

or

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}(kos\frac{\theta + 2\pi k}{n} + j\sin\frac{\theta + 2\pi k}{n}) \text{ if } \theta \text{ in radians}$$

for $k = 0, 1, 2, ..., n – 1$.

Example 18

Find the cube roots of $z = 8j$.

Solution:

$z = 8j$ or in polar form $z = 8(\cos 90^{o} + j \sin 90^{o})$ then

$$z^{\frac{1}{3}} = 8^{\frac{1}{3}}(\cos\frac{90^{o}+360^{o}k}{3} + j\sin\frac{90^{o}+360^{o}k}{3}) \quad \text{for } k = 0, 1, 2.$$

when $k = 0$

$$z^{\frac{1}{3}} = 2(\cos 30^{o} + j \sin 30^{o})$$
$$= \sqrt{3} + j$$

when $k = 1$

$$z^{\frac{1}{3}} = 2(\cos 150^{o} + j \sin 150^{o})$$
$$= -\sqrt{3} + j$$

when $k = 2$

$$z^{\frac{1}{3}} = 2(\cos 270^{o} + j \sin 270^{o})$$
$$= -2j$$

Therefore the roots of $z = 8j$ are $\sqrt{3} + j$, $-\sqrt{3} + j$ and $-2j$.

2.8 EXPANSIONS FOR COS^n AND SIN^n IN TERMS OF COSINES AND SINES OF MULTIPLES OF θ

Definition 2.13:
If $z = \cos\theta + j\sin\theta$ then

i) $z^{n} + \dfrac{1}{z^{n}} = 2\cos n\theta$

ii) $z^{n} - \dfrac{1}{z^{n}} = 2j\sin n\theta$

Definition 2.14: Binomial Theorem
If $n \in N$, then

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots + {}^nC_r a^{n-r} b^r + \ldots + {}^nC_{n-1} a b^{n-1} + {}^nC_n b^n$$

with ${}^nC_r = \dfrac{n!}{r!(n-r)!}$.

Example 19

State $\sin^5\theta$ in terms of sines

Solution:

Using theorem 4, $z = \cos\theta + j\sin\theta$, then

$$z - \frac{1}{z} = 2j\sin\theta$$

$$(2j\sin\theta)^5 = [z - \frac{1}{z}]^5.$$

By using Binomial expansion, then

$$[z - \frac{1}{z}]^5 = z^5 - 5z^4\left(\frac{1}{z}\right) + 10z^3\left(\frac{1}{z}\right)^2 - 10z^2\left(\frac{1}{z}\right)^3 + 5z\left(\frac{1}{z}\right)^4 - \left(\frac{1}{z}\right)^5$$

$$= \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$$

Thus,

$$32j\sin^5\theta = 2j\sin 5\theta - 5(2j\sin 3\theta) + 10(2j\sin\theta)$$

$$\sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$$

2.9 LOCI IN THE COMPLEX NUMBERS

> **Definition 2.15:**
> A locus in a complex plane is the set of points that have a specified property. A locus of a point in a complex plane could be a straight line, cirle, ellipse and etc.

Example 20

If $z = a + bj$, find the equation of the locus defined by:

i) $\left|\dfrac{z-2j}{z-1}\right| = 1$ ii) $|z-(2+3j)| = 2$

Solution:

i) $\left|\dfrac{z-2j}{z-1}\right| = \left|\dfrac{a+bj-2j}{a+bj-1}\right| = 1$

$$|a+bj-2j| = |a+bj-1|$$

$$|a+(b-2)j| = |a-1+bj|$$

$$\sqrt{a^2+(b-2)^2} = \sqrt{(a-1)^2+b^2}$$

$$a^2+b^2-4b+4 = a^2-2a+1+b^2$$

$$2a-4b+3=0$$

The locus is a straight line with slope $\dfrac{1}{2}$.

ii) $|z-(2+3j)| = |a+bj-2-3j| = 2$

$$|a-2+(b-3)j| = 2$$

$$\sqrt{(a-2)^2+(b-3)^2} = 2$$

$$(a-2)^2+(b-3)^2 = 2^2$$

The locus is a circle of radius 2 and centre (2,3).

Tutorial 2

1. Simplify the following (addition and subtraction):

   a) $(3+j)+(1+2j)$
   b) $(5-3j)+(4+3j)$
   c) $(2-3j)-(1+2j)$
   d) $(1+j)-(1-j)$

2. Simplify the following (multiplication):

   a) $(2+3j)(4+5j)$
   b) $(2-j)(3+2j)$
   c) $(1+j)(1-j)$
   d) $(3+4j)(3-4j)$
   e) $(\mathrm{u}+vj)(\mathrm{u}-vj)$,
   f) $(\mathrm{x}+2yj)(2\mathrm{x}+\mathrm{y}j)$
   g) $j(2p+3pj)$
   h) $(\mathrm{p}+2qj)(\mathrm{p}-2\mathrm{q}j)$

3. Simplify the following (division):

   a) $\dfrac{1-j}{1+j}$
   b) $\dfrac{1}{2-3j}$
   c) $\dfrac{3j-2}{1+2j}$
   d) $\dfrac{5+4j}{5-4j}$
   e) $\dfrac{1}{x+yj}$
   f) $\dfrac{1}{x-yj}$
   g) $\dfrac{1}{2+3j}+\dfrac{1}{2-3j}$

4. Simplify the following:

   a) $(4-5j)^2$
   b) $(1-j)^3$
   c) $\dfrac{1}{(1+j)^3}$,
   d) $(\cos\theta+j\sin\theta)(\cos\theta-j\sin\theta)$

5. Find the value of *a* and *b*.

a) if $(\mathrm{a}+\mathrm{b})+j(\mathrm{a}-\mathrm{b})=(2+5j)^2+j(2-3j)$

b) Given that, $(\mathrm{a}+\mathrm{b})+j(\mathrm{a}-\mathrm{b})=(1+j)^2+j(2+j)$

6. Evaluate the following

a) $(2+3j)(6+7j)$

b) $(2+3j)(2-3j)$

c) $\dfrac{4+3j}{2-j}$

7. Write in the simplest form:

a) $(5+4j)(3+7j)(2-3j)$

b) $\dfrac{(2-3j)(3+2j)}{(4-3j)}$

8. State the following in the form of $a+bj$.

a) $\dfrac{2+3j}{j(4-5j)}+\dfrac{2}{j}$

b) $\dfrac{1}{2+3j}+\dfrac{1}{1-2j}$

9. If $z=\dfrac{2+j}{1-j}$, state $z+\dfrac{1}{z}$ in complex number.

10. Find the complex number which satisfies the equation:

$$3z\bar{z}+2(z-\bar{z})=39+12j$$

11. Label the following complex numbers on Argand Diagram.

a) $2+j$

b) $-1+2j$

c) $3j$

12. Find the modulus and argument of the following complex numbers:

a) $3+4j$

b) $4j$

c) $-2-\sqrt{5}j$

d) $5-12j$,

e) $-1-2j$,

13. State the following in polar form:

a) $5+5j$

b) $-6+3j$

c) $(4-7j)(2+3j)$

d) $\dfrac{4+3j}{2-j}$

14. State the following in exponential form:

a) $-1-j$

b)$1-\sqrt{3}j$

c) $4(\cos 30^\circ + j\sin 30^\circ)$

Answers

1. a) $(4+3j)$

   b) 9

   c) $(1-5j)$

   d) $2j$

2. a) $-7+22j$

   b) $8+j$

   c) 2

   d) 25

   e) u^2+v^2,

   f) $2(x^2-y^2)+5xyj$,

   g) $-3p+2pj$,

   h) p^2+4q^2

3. a) $-j$

b) $\frac{2}{13}+\frac{3}{13}j$

c) $\frac{4}{5}+\frac{7}{5}j$

d) $\frac{9}{41}+\frac{40}{41}j$

e) $\frac{x}{x^2+y^2}-\frac{y}{x^2+y^2}j$

f) $\frac{x}{x^2+y^2}+\frac{y}{x^2+y^2}j$

g) $\frac{4}{13}$

4. a) -9 - 40j

b) -2 - 2j

c) $\frac{-1-j}{4}$

d) 1

5. a) $a=2; b=-20$

b) $a=\frac{3}{2}; b=-\frac{5}{2}$

6. a) $-9+32j$

b) 13

c) $1+2j$

7. a) $115+133j$

b) $\frac{63}{25}+\frac{16}{25}j$

8. a) $\frac{22}{41}-\frac{75}{41}j$

b) $\frac{23}{65}+\frac{11}{65}j$

9. $\frac{7}{10}+\frac{9}{10}j$

10. $x=\pm 2; y=3$

11. a) $(2,1)$

b) $(-1,2)$

c) $(0,3)$

12. a) $|z| = 5, \theta = \tan^{-1}\left(\frac{4}{3}\right)$

b) $|z| = 4, \theta = \tan^{-1}(\infty) = \frac{\pi}{2}$

c) $|z| = 3, \theta = \tan^{-1}\left(\frac{\sqrt{5}}{2}\right)$

d) $|z| = 13, \theta = \tan^{-1}\left(\frac{-12}{5}\right)$

e) $|z| = \sqrt{5}, \theta = \tan^{-1}(2),$

13. a) $z = 5\sqrt{2}(\cos 45^0 + j\sin 45^0)$

b) $z = 3\sqrt{5}(\cos 153.4^0 + j\sin 153.4^0),$

c) $z = \sqrt{31}(\cos 356^o + j\sin 356^o)$

d) $z = \sqrt{5}(\cos 63^o + j\sin 63^o),$

14. a) $z = \sqrt{2}e^{\frac{5\pi}{4}j}$

b) $z = 2e^{\frac{5\pi}{2}j}$

c) $z = 4e^{\frac{\pi}{6}j}$

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