CHAPTER 1:
First Order Differential Equations: Types of Equations

1) Solve the following differential equation by using method of separation variables.

$$\frac{dy}{dx}=\frac{(y-1)(y+1)}{x}$$

$$Ans:\ \left(\frac{y-1}{y+1}\right)=Ax^2$$

2) Solve the following differential equation by using homogeneous equation method.

$$\frac{dy}{dx}=\frac{3y^2+4xy}{2xy+2x^2}$$

$$Ans:\ y=\frac{Ax^3}{y+2x}$$

3) Solve the following differential equation by using linear equation method.

$$(x-3)\frac{dy}{dx}-y=(x-3)^2, \quad y(4)=10$$

$$Ans:\ y=(x+6)(x-3)$$

4) Solve the following differential equation by using exact equation method.

$$(2xy+4y)\,dy+\left(4x^3+y^2+2\right)dx=0$$

$$Ans:\ x^4+xy^2+2x+2y^2=B \ \ where\ \ B=C-A$$

Application of First Order Differential Equation

1) According to Forestry Department of Peninsular Malaysia, the number of forest herbs population in Pahang in 2012 was 0.84 million. In 2014, the population was 0.87 million. Assume that the population of forest herbs in Pahang is growing, estimate the population in 2018.

Ans: The forest herbs population in Pahang for 2018 is 0.93 million

2) According to Zoo Taiping Malaysia, the number of tiger population 2 months ago was 3 tigers and now increase to 18 tigers. Assume that the population of tiger is growing, estimate the population in 2 month and 1 year from now.

$$\textit{Ans: } P(4) = 108.0088 \approx 108 \textit{ tigers}$$
$$P(14) = 840,046 \textit{ tigers}$$

3) The half-life of toxic in your body is about 3 hours. How much the toxic left after 6 hours?

$$\textit{Ans: } P(6) = 0.2501 \approx 0.25$$

CHAPTER 2:
Second Order Linear Differential Equations: Homogeneous Equations

1) Find the solutions of differential equations :

a) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 0,\ \ y(0) = 2\ \ y'(0) = 1$ $\qquad Ans:\ y = \dfrac{1}{4}(5e^{-x} + 3e^{3x})$

b) $\dfrac{d^2y}{dx^2} - 8\dfrac{dy}{dx} + 17y = 0,\ \ y(0) = -4,\ y'(0) = -1$ $\qquad Ans:\ y = e^{4x}(-4\cos x + 15\sin x)$

Second Order Linear Differential Equations: Nonhomogeneous Equations

2) By using the method of undetermined coefficient, solve the following differential equation.

a) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 2x^2$ $\qquad Ans:\ y = (A + Bx)e^{2x} + \dfrac{1}{2}x^2 + x + \dfrac{3}{4}$

b) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = 4x - 6,\ \ y(0) = 3,\ y'(0) = -4$ $\qquad Ans:\ y = e^{-2x} - 2x + 2$

CHAPTER 3:
Laplace Transform

1) Find the Laplace transform of the following functions.

a) $f(t) = 3 + t^2$ *Ans:* $F(s) = \frac{3}{s} + \frac{2}{s^3}$

b) $f(t) = t - 4$ *Ans:* $F(s) = \frac{1}{s^2} - \frac{4}{s}$

c) $f(t) = 2t^3 + 4t^2 - 9$ *Ans:* $F(s) = \frac{12}{s^4} + \frac{8}{s^3} - \frac{9}{s}$

d) $f(t) = e^{-2t}\cos 4t$ *Ans:* $F(s) = \frac{(s+2)}{s^2 + 4s + 20}$

2) Find the inverse Laplace transforms of the following functions.

a) $F(s) = \frac{2s+5}{(s+6)(s-7)}$ *Ans:* $f(t) = \frac{7}{13}e^{-6t} + \frac{19}{13}e^{7t}$

b) $F(s) = \frac{5}{(s-1)^2 - 16}$ *Ans:* $f(t) = \frac{5e^t}{4}\sinh 4t$

c) $F(s) = \frac{7}{(s+3)^2 + 9}$ *Ans:* $f(t) = \frac{7e^{-3t}}{3}\sin 3t$

d) $F(s) = \frac{1}{(s-1)^2 + 49}$ *Ans:* $f(t) = \frac{e^t}{7}\sin 7t$

e) $F(s) = \frac{s}{(s-5)^2 - 4}$ *Ans:* $f(t) = e^{5t}\cosh 2t$

f) $F(s) = \frac{s-1}{(s+3)^2 + 25}$ *Ans:* $f(t) = e^{-3t}\cos 5t - \frac{4}{5}e^{-3t}\sin 5t$

Laplace Transforms: Initial Value Problems

1) Given the following initial value problem,

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=0\,; \qquad y(0)=4; \qquad y'(0)=9\,.$$

a) Show the Laplace transforms for the initial value problem above is given by

$$Y(s)=\frac{4s+29}{(s+2)(s+3)}\,.$$

b) By using part 2a), solve the initial value problem using the Laplace transforms method.

Ans: $y(t)=21e^{-2t}-17e^{-3t}$

Table of Laplace Transforms

$f(t)$	$\mathcal{L}\{f(t)\}=F(s)$	$f(t)$	$\mathcal{L}\{f(t)\}=F(s)$
a	$\frac{a}{s}$	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$t^n,\ n=1,2,3,..$	$\frac{n!}{s^{n+1}}$	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
e^{at}	$\frac{1}{s-a}$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin at$	$\frac{a}{s^2+a^2}$	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$
$\cos at$	$\frac{s}{s^2+a^2}$	$e^{at} f(t)$	$F(s-a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y'(t)$	$sY(s)-y(0)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

First Shift Property of Inverse laplace transform	$L^{-1}\{F(s-a)\}=e^{at}L^{-1}\{F(s)\}$ $=e^{at}f(t)$