CHAPTER 1:
First Order Differential Equations: Types of Equations

1) Solve the following differential equation by using method of separation variables.

$$\frac{dy}{dx}=\frac{y}{(x-1)(x+1)}$$

Ans: $y=A\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$

2) Solve the following differential equation by using homogeneous equation method.

$$2x^2\frac{dy}{dx}=y^2+x^2$$

Ans: $\frac{-2x}{(y-x)}=\ln x+C$

3) Solve the following differential equation by using linear equation method.

$$\frac{dy}{dx}+3y=e^{4x}$$

Ans: $y=\frac{e^{3x}}{7}+Ce^{-3x}$

4) Solve the following differential equation by using exact equation method.

$$\left(2x^2y-\frac{1}{2}y\right)dy+\left(2xy^2+4\right)dx=0$$

Ans: $x^2y^2+4x-\frac{y^2}{4}=B \;\; where \;\; B=C-A$

Application of First Order Differential Equation

1) In year 2000, the population of crocodile in Malaysia was estimated as 200. After ten years, the population has grown to be 500. Estimate the population of the crocodile in year 2017.

Ans: 949

2) In 1990, the Department of Natural Resources releases 1000 fish into a lake. In 1997, the population of the fish was estimated to be 3000. By assuming no other factor affect the population, estimate the number of fish in year 2000.

Ans: 4803

3) The half-life of Uranium-232 is 68.9 years. How much of a 100g sample is present after 250 years?

Ans: $P(250) = 8.0058$

CHAPTER 2:
Second Order Linear Differential Equations: Homogeneous Equations

1) Find the solutions of differential equations :

a) $\frac{d^2y}{dx^2} + 9\frac{dy}{dx} = 0, \quad y(0) = 2,\ y'(0) = -3$ $\qquad Ans: \ y = 2\cos(3x) - \sin(3x)$

b) $16\frac{d^2y}{dx^2} - 40\frac{dy}{dx} + 25y = 0, \quad y(0) = 3,\ y'(0) = -\frac{9}{4}$ $\qquad Ans: \ y = (3-6x)e^{\frac{5}{4}x}$

Second Order Linear Differential Equations: Nonhomogeneous Equations

2) By using the method of undetermined coefficient, solve the following differential equation.

a) $2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 3y = 5x + 1, \quad y(0) = 3,\ y'(0) = 1$ $\qquad Ans: \ y = \frac{1}{9}e^{-3x} + 6e^{\frac{1}{2}x} - \frac{5}{3}x - \frac{28}{9}$

b) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2x^2$ $\qquad Ans: \ y = (A + Bx)e^{2x} + \frac{1}{2}x^2 + x + \frac{3}{4}$

CHAPTER 3:
Laplace Transform

1) Find the Laplace transform of the following functions.

a) $f(t)=e^{3t}\sin 6t$ 　　*Ans:* $F(s)=\dfrac{6}{s^2-6s+45}$

b) $f(t)=e^{-2t}\cosh 7t$ 　　*Ans:* $F(s)=\dfrac{(s+2)}{s^2+4s-45}$

c) $f(t)=e^{4t}\sinh 8t$ 　　*Ans:* $F(s)=\dfrac{8}{s^2-8s-48}$

d) $f(t)=t\sin 3t$ 　　*Ans:* $F(s)=\dfrac{6s}{(s^2+9)^2}$

2) Find the inverse Laplace transforms of the following functions.

a) $F(s)=\dfrac{5s}{s^2+9}$ 　　*Ans:* $f(t)=5\cos 3t$

b) $F(s)=\dfrac{3+s}{s^2-16}$ 　　*Ans:* $f(t)=\dfrac{3}{4}\sinh 4t+\cosh 4t$

c) $F(s)=\dfrac{s-10}{s^2+25}$ 　　*Ans:* $f(t)=\cos 5t+2\sin 5t$

d) $F(s)=\dfrac{s+5}{s^2-4}$ 　　*Ans:* $f(t)=\cosh 2t+\dfrac{5}{2}\sinh 4t$

e) $F(s)=\dfrac{s+1}{(s-4)(s+3)}$ 　　*Ans:* $f(t)=\dfrac{5}{7}e^{4t}+\dfrac{2}{7}e^{-3t}$

Laplace Transforms: Initial Value Problems

1) Given the following initial value problem,

$$y''+y'-6y=5 \quad ; \quad y(0)=2; \quad y'(0)=3.$$

a) Show the Laplace transforms for the initial value problem above is given by

$$Y(s)=\frac{2s^2+5s+5}{s(s+3)(s-2)}.$$

b) By using part 1a), solve the initial value problem using the Laplace transforms method.

Ans: $y(t) = -\frac{5}{6} + \frac{8}{15}e^{-3t} + \frac{23}{10}e^{2t}$

Table of Laplace Transforms

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$	$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
a	$\frac{a}{s}$	$e^{at}\sin bt$	$\frac{b}{(s-a)^2 + b^2}$
$t^n,\ n = 1,2,3,..$	$\frac{n!}{s^{n+1}}$	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2 + b^2}$
e^{at}	$\frac{1}{s-a}$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin at$	$\frac{a}{s^2 + a^2}$	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$
$\cos at$	$\frac{s}{s^2 + a^2}$	$e^{at} f(t)$	$F(s-a)$
$\sinh at$	$\frac{a}{s^2 - a^2}$	$y'(t)$	$sY(s) - y(0)$
$\cosh at$	$\frac{s}{s^2 - a^2}$	$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$

First Shift Property of Inverse laplace transform	$L^{-1}\{F(s-a)\} = e^{at}L^{-1}\{F(s)\}$ $= e^{at} f(t)$