CHAPTER 1:
First Order Differential Equations: Types of Equations

1) Solve the following differential equation by using method of separation variables.

$$\frac{x+4}{y^2+3}=x^2y\frac{dy}{dx}$$

Ans: $\frac{y^5}{5}+\frac{3y^2}{2}=\ln x-\frac{4}{x}+C$

2) Solve the following differential equation by using homogeneous equation method.

$$\frac{dy}{dx}=\frac{y^2}{xy-4x^2}$$

Ans: $y^4=Ae^{\frac{y}{x}}$

3) Solve the following differential equation by using linear equation method.

$$x\frac{dy}{dx}+y=xe^x$$

Ans: $y=e^x-\frac{e^x}{x}+\frac{C}{x}$

4) Solve the following differential equation by using exact equation method.

$$\left(3x^2y-2y\right)\frac{dy}{dx}=-\left(3xy^2+2x\right)$$

Ans: $\frac{3x^2y^2}{2}+x^2-y^2=B$ *where* $B=C-A$

Application of First Order Differential Equation

1) A population growth and decay is given by the equation, $\frac{dP}{dt}=kP(t)$ where $P(t)$ is the population at time t and k is a constant. Show that the general solution to the equation is $P(t)=Ae^{kt}$ where A is a constant.

2) Around thousands citizens are living in a small country and this amount increased to 450 thousands within 2 years. The population growth at rate that is proportional to the number of the citizen at that time. Assume that initial citizen is 50 thousand, determine the number of citizen after 5 years.

Ans: 12150

3) A biologist discovered a new species of bacteria. At time $t=0$, they put 100 bacteria into what he has determined to be a favorable growth medium. Ten hour leter, they had identified 450 bacteria exist inside the growth medium. Estimate the population of the bacteria after 25 hours and 40 hours.

Ans: 41006

CHAPTER 2:
Second Order Linear Differential Equations: Homogeneous Equations

1) Find the solutions of differential equations :

a) $3\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=0,\ \ y(0)=1,\ y'(0)=-1$ $\quad Ans:\ y=-2+3e^{-\frac{1}{3}x}$

b) $16\dfrac{d^2y}{dx^2}-40\dfrac{dy}{dx}+25y=0,\ \ y(0)=3,\ y'(0)=-\dfrac{9}{4}$ $\quad Ans:\ y=(3-6x)e^{\frac{5}{4}x}$

Second Order Linear Differential Equations: Nonhomogeneous Equations

2) By using the method of undetermined coefficient, solve the following differential equation.

a) $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+5y=2e^{-2x},\ \ y(0)=1,\ y'(0)=-2$ $\quad Ans:\ y=e^{-2x}(2-\cos x)$

b) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+2y=5x+1,\ \ y(0)=1,\ y'(0)=2$ $\quad Ans: y=e^{-x}(3\cos x+\dfrac{5}{2}\sin x)+\dfrac{5}{2}x-2$

CHAPTER 3:
Laplace Transform

1) Find the Laplace transform of the following functions.

a) $f(t) = t\sinh 2t$ *Ans:* $F(s) = \dfrac{4s}{\left(s^2-4\right)^2}$

b) $f(t) = t\cos 5t$ *Ans:* $F(s) = \dfrac{s^2-25}{\left(s^2+25\right)^2}$

c) $f(t) = 2te^{4t}$ *Ans:* $F(s) = \dfrac{2}{(s-4)^2}$

d) $f(t) = 5t^2e^{-t}$ *Ans:* $F(s) = \dfrac{10}{(s+1)^3}$

2) Find the inverse Laplace transforms of the following functions.

a) $F(s) = \dfrac{4}{s^3}$ *Ans:* $f(t) = 2t^2$

b) $F(s) = \dfrac{1}{s^7}$ *Ans:* $f(t) = \dfrac{1}{720}t^6$

c) $F(s) = -\dfrac{2}{s^6}$ *Ans:* $f(t) = -\dfrac{1}{60}t^5$

d) $F(s) = \dfrac{2s}{s^2-49}$ *Ans:* $f(t) = 2\cosh 7t$

e) $F(s) = \dfrac{3}{s^2+64}$ *Ans:* $f(t) = \dfrac{3}{8}\sin 8t$

Laplace Transforms: Initial Value Problems

1) Use the method of Laplace Transforms to find the solution of the following initial value problems.

a) $\dfrac{dy}{dt}+2y=0 \quad ; \quad y(0)=2$ *Ans:* $y(t)=2e^{-2t}$

b) $y'-y=e^{-t} \quad ; \quad y(0)=-1$ *Ans:* $y(t)=-\frac{1}{2}e^{t}-\frac{1}{2}e^{-t}$

c) $y'+y=1 \quad ; \quad y(0)=1$ *Ans:* $y(t)=1$

d) $2y'+y=t \quad ; \quad y(0)=2$ *Ans:* $y(t)=-2+t+4e^{-\frac{1}{2}t}$

Table of Laplace Transforms

$f(t)$	$\mathcal{L}\{f(t)\}=F(s)$	$f(t)$	$\mathcal{L}\{f(t)\}=F(s)$
a	$\frac{a}{s}$	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$t^n,\ n=1,2,3,..$	$\frac{n!}{s^{n+1}}$	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
e^{at}	$\frac{1}{s-a}$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin at$	$\frac{a}{s^2+a^2}$	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$
$\cos at$	$\frac{s}{s^2+a^2}$	$e^{at} f(t)$	$F(s-a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y'(t)$	$sY(s)-y(0)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

First Shift Property of Inverse laplace transform	$L^{-1}\{F(s-a)\}=e^{at}L^{-1}\{F(s)\}$ $=e^{at}f(t)$