
UNIVERSITI MALAYSIA PERLIS

Peperiksaan Akhir Semester Pertama
Sidang Akademik 2018/2019

Oktober 2018

DQT203 – Mathematics III
[Matematik III]

Masa : 3 jam

Please make sure that this question paper has **SIX (6)** printed pages including this front page before you start the examination.
*[Sila pastikan kertas soalan ini mengandungi **ENAM (6)** muka surat yang bercetak termasuk muka hadapan sebelum anda memulakan peperiksaan ini.]*

This question paper has **FOUR (4)** questions. Answer **ALL** questions.
*[Kertas soalan ini mengandungi **EMPAT (4)** soalan. Jawab **SEMUA** soalan.]*

Question 1

[Soalan 1]

(a) Solve the following differential equation by using an appropriate method.
[Selesaikan persamaan pembezaan berikut menggunakan kaedah yang bersesuaian.]

$$\frac{dy}{dx} = \frac{\left(y^2 - 1\right)}{x}$$

(10 Marks/ *Markah*)

(b) Given the following differential equation
[Diberi persamaan pembezaan berikut]

$$\left(\cos x - \sin x + y^2\right)dx + \left(2xy\right)dy = 0.$$

Show that the given differential equation is exact. Subsequently, solve the above differential equation.
[Tunjukkan bahawa persamaan yang diberi adalah persamaan tepat. Seterusnya, selesaikan persamaan pembezaan di atas.]

(10 Marks/ *Markah*)

Question 2

[Soalan 2]

(a) A Newton's Law of Cooling is given by the equation,

[Hukum penyejukan Newton diberikan oleh persamaan.]

$$\frac{dT}{dt} = -k(T - T_s)$$

where T is the temperature of the object, T_s is the surrounding temperature and k is a constant of proportionality. Show that the general solution to the equation is $T = Ae^{-kt} + T_s$ where A is a constant.

[di mana T adalah suhu sesuatu objek, T_s adalah suhu persekitaran dan k adalah pemalar perkadaran. Tunjukkan bahawa penyelesaian umum bagi persamaan tersebut adalah $T = Ae^{-kt} + T_s$ di mana A adalah pemalar.]

(5 Marks/ *Markah*)

(b) A pizza is removed from an oven with temperature of 350° F and placed to cool in a kitchen with temperature 75° F. After 15 minutes, the pizza has a temperature of 150° F. Determine the time required to cool the pizza to temperature of 80° F.

[Sekeping piza dikeluarkan dari sebuah ketuhar dengan suhu 350° F dan dibiarkan menyejuk dalam sebuah dapur dengan suhu 75° F. Selepas 15 minit, suhu piza tersebut adalah 150° F. Tentukan masa yang diperlukan untuk menyejukkan piza itu ke suhu 80° F.]

(7 Marks/ *Markah*)

(c) The temperature of a dead body when it was found at 3 o'clock in the morning is 85° F. The surrounding temperature at that time was 68° F. After 2 hours, the temperature of the dead body decreased to 74° F. Assuming that the normal body temperature is 98.6° F, determined the time of murdered.

[Suhu satu mayat ketika dijumpai pada pukul 3 pagi ialah 85° F. Suhu persekitaran pada ketika itu ialah 68° F. Selepas 2 jam, suhu mayat tersebut menurun kepada 74° F. Dengan menganggap suhu badan normal ialah 98.6° F, tentukan waktu pembunuhan.]

(8 Marks/ *Markah*)

Question 3
[Soalan 3]

(a) Find the particular solution of the following differential equation.
[Cari penyelesaian khusus bagi persamaan pembezaan berikut.]

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0, \quad y(0)=2,\ y'(0)=4$$

(10 Marks/ *Markah*)

(b) Given the following differential equation,
[Diberi persamaan pembezaan berikut.]

$$y''-4y'+4y=\frac{12e^{2x}}{x^4}$$

(i) Find y_1 and y_2.
[Cari y_1 dan y_2.]

(4 Marks/ *Markah*)

(ii) Find the Wronskian (W).
[Cari Wronskian (W).]

(5 Marks/ *Markah*)

(iii) Find u and v.
[Cari u dan v.]

(8 Marks/ *Markah*)

(iv) Express the general solution for the following equation.
[Nyatakan penyelesaian umum bagi persamaan berikut.]

(3 Marks/ *Markah*)

Question 4

[Soalan 4]

(a) Find the Laplace transform of the following functions.
[Dapatkan jelmaan Laplace bagi fungsi-fungsi berikut.]

(i) $f(t) = -10 - 2t^2$

(ii) $f(t) = e^{-t}\cosh 3t$

(iii) $f(t) = t\sin t$

(10 Marks/ *Markah*)

(b) Find the inverse Laplace transforms of the following functions.
[Dapatkan songsangan jelmaan Laplace bagi fungsi-fungsi berikut.]

(i) $F(s) = \dfrac{4}{s^5}$

(ii) $F(s) = \dfrac{2}{s^2 - 49}$

(iii) $F(s) = \dfrac{4s}{4s^2 + 9}$

(8 Marks/ *Markah*)

(c) Given the following initial value problem:
[Diberi masalah nilai awal berikut:]

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 24y = 24, \quad y(0) = 1\ ,\ y'(0) = 5.$$

(i) Show that the Laplace transform for the initial value problem above is given by
[Tunjukkan bahawa jelmaan Laplace bagi masalah nilai awal berikut diberikan oleh]

$$Y(s) = \frac{s^2 + 7s + 24}{s(s+6)(s-4)}.$$

(5 Marks/ *Markah*)

(ii) By using $Y(s)$ as in part c(i), solve the following initial value problem by using Laplace transforms method.
[Dengan menggunakan $Y(s)$ seperti pada bahagian c(i), selesaikan masalah nilai awal berikut dengan menggunakan kaedah jelmaan Laplace.]

(7 Marks/ *Markah*)

....6/-

APPENDIX
[LAMPIRAN]

LIST OF FORMULAS
[SENARAI FORMULA]

Table of Laplace Transforms

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$	$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
a	$\frac{a}{s}$	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$t^n,\ n=1,2,3,..$	$\frac{n!}{s^{n+1}}$	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
e^{at}	$\frac{1}{s-a}$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin at$	$\frac{a}{s^2+a^2}$	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$
$\cos at$	$\frac{s}{s^2+a^2}$	$e^{at} f(t)$	$F(s-a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y'(t)$	$sY(s)-y(0)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

Method of Undetermined Coefficients

$$y_p(x) = x^r\left(Cx^n + Dx^{n-1} + Ex^{n-2} + \ldots\right),\ r = 0,1,2,\ldots$$

$$y_p(x) = x^r\left(Ce^{kx}\right),\ r = 0,1,2,\ldots$$

$$y_p(x) = x^r\left[C\cos(kx) + D\sin(kx)\right],\ r = 0,1,2,\ldots$$

Method of Variation Parameters

$$y = y_h(x) + y_p(x),\ y_p(x) = uy_1(x) + vy_2(x)$$

$$u = -\int \frac{y_2(x)f(x)}{aW}dx, \quad v = \int \frac{y_1(x)f(x)}{aW}dx$$

$$W = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix}$$

-oooOooo-