UNIVERSITI MALAYSIA PERLIS

MIDTERM EXAMINATION
DQT 203 (Mathematics III)
16 January 2020 (Thursday)
8.30 - 9.30 PM

Lecturer Names:
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Answer all questions. All working steps must be shown clearly in the answer booklet.

1. Solve the following differential equations and write your answer in the simplest form:

   (a) Given $xdy - y^2 dx = 0$, find the general solution using separable method.

   (4 marks)

   (b) Given $\frac{dy}{dx} + 4y = e^{3x}$, find the general solution using linear equation.

   (6 marks)

2. Given a differential equation, $xy^2 dx = -x^2 y dy$.

   (a) Show the equation is an exact equation.

   (3 marks)

   (b) Find the general solution of the equation.

   (7 marks)

3. Find the particular solution of the following differential equation,

   $$y'' + 25y = 0,\ y(0) = 1 \text{ and } y'(0) = -5.$$

   (10 marks)

ooOoo

No. Soalan:.....
(Question No: 1(a).....)

(10)

Jangan tulis di atas ruangan ini (Do not write in this column)

1. (a) $x\,dy - y^2\,dx = 0$ (using separable)

$x\,dy - y^2\,dx = 0$

$x\,dy = y^2\,dx$ ✓

$\int \frac{1}{y^2}\,dy = \int \frac{1}{x}\,dx$

$\int y^{-2}\,dy = \int \frac{1}{x}\,dx$ ✓

$\frac{-y^{-1}}{1} = \ln x + c$

$-\frac{1}{y} = \ln x + c$ ✓

$\frac{1}{y} = -\ln x - c$ ✓

$y^{-1} = -(\ln x + c) \Rightarrow$

No Soalan
(Question No. 1(b))

b) Given $\frac{dy}{dx} + 4y = e^{3x}$, y is a linear

$\frac{dy}{dx} + 4y = e^{3x}$

$p(x) = 4$, $\theta(x) = e^{3x}$ 1

$I = e^{\int 4\,dx}$

$= e^{4x}$ 2

$I(y) = \int \theta(x) I \, dx$

$e^{4x}(y) = \int e^{3x} \cdot e^{4x} \, dx$

$e^{4x}(y) = \int e^{7x} \, dx$

$e^{4x}(y) = \frac{e^{7x}}{7} + C$ 1

$e^{4x}(y) = \frac{1}{7}(e^{7x}) + C$

$y = \frac{1}{7}\left(\frac{e^{7x}}{e^{4x}}\right) + \frac{C}{e^{4x}}$

$y = \frac{1}{7}(e^{3x}) + \frac{C}{e^{4x}}$ 1

$= \frac{1}{7}(e^{3x}) + (e^{-4x}$ #

No. Soalan
(Question No. 2(a))

(10)

2 $x y^2 \, dx = -x^2 y \, dy$

(a) Show the equation is an exact equation

$M \, dx + N \, dy = 0$ (exactness equation)

$x y^2 \, dx = -x^2 y \, dy$

$x y^2 \, dx + x^2 y \, dy = 0$

$M \, dx + N \, dy = 0$

$M = x y^2$ 1 , $N = x^2 y$ 1

$\frac{\partial M}{\partial y} = 2xy$ 1 , $\frac{\partial N}{\partial x} = 2xy$ 1

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ 1

(Exactness is proved)

No. Soalan
(Question No. 2(b))

(b) Find the general solution of the equation

① $F = \int N \, dx + \phi(y)$

$= \int x y^3 \, dx + \phi(y)$

$= \frac{x^2}{2} y^3 + \phi(y)$

② $\frac{\partial F}{\partial y} = x^2 y + \phi'(y)$

$\frac{\partial F}{\partial y} = N$

$x^2 y = x^2 y + \phi'(y)$

$\phi'(y) = 0$

③ $\phi(y) = \int \phi'(y) \, dy$

$= \int 0 \, dy$

$= 0$

④ $\frac{x^2 y^2}{2} + 0 = c$

$\frac{x^2 y^2}{2} = c$

No Soalan /Question No 3 (10)

3 $y'' + 25y = 0$, $y(0) = 1$, $y'(0) = -5$

$m^2 + 25 = 0$ ✓1

$m^2 = -25$

$m = \sqrt{-25}$ ✓1

$m = \sqrt{(25)(-1)}$

$m = 5i$

$\alpha = 0$, $\beta = 5$ ✓1

$y(x) = e^{\alpha x}(A \cos \beta x + B \sin \beta x)$

$= e^{0x}(A \cos 5x + B \sin 5x)$

$= A \cos 5x + B \sin 5x$ ✓1

$y'(x) = -5A \sin 5x + 5B \cos 5x$ ✓1

$y(x) = A \cos 5x + B \sin 5x$

$y(0) = 1$

$1 = A \cos 5(0) + B \sin 5(0)$

$1 = A$ ✓2

$A = 1$

$y'(x) = -5A \sin 5x + 5B \cos 5x$

$y'(0) = -5$

$-5 = -5A \sin 5(0) + 5B \cos 5(0)$

$-5 = 0 + 5B$

$-5 = 5B$ ✓2

$B = -1$ ✓1

$y(x) = \cos 5x - \sin 5x$ //