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NMJ 20703 - SIGNALS AND SYSTEMS
ASSIGNMENT 2

1. Calculate the trigonometric fourier series for the full wave rectifier sine wave shown in figure 1.

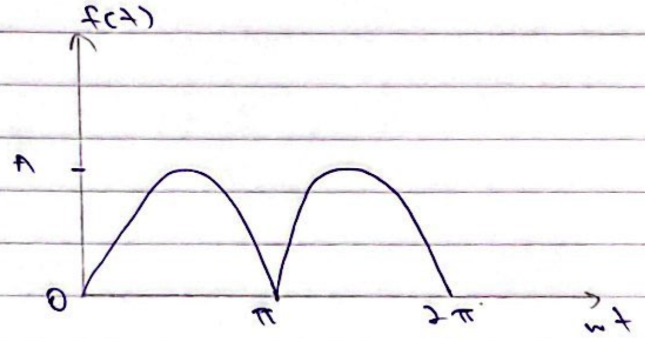

Figure 1

Answer:

$f(t) = A\,|\sin t|$

Even, $b_n = 0$ Odd, $a_n = 0$

$$a_0 = \frac{1}{T}\int_{-T}^{T} f(t)\,.\,dt$$

$$= \frac{2}{\pi}\int_0^{\pi} A\sin(\omega t)\,dt$$

$$= \frac{2A}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$= \frac{2A}{\pi}\left[-(-1)+1\right]$$

$$a_0 = \frac{4A}{\pi} \;\#$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(t)\cdot \cos n(\omega t)\, d\omega t$$

$$= \frac{2}{\pi}\int_0^{\pi} A \sin(\omega t)\cos(\omega t)\, d\omega t$$

$$= \frac{2A}{2\pi}\left[\frac{-\cos(1+n)\omega t}{1+n} + \frac{-\cos(1-n)\omega t}{1-n}\right]_0^{\pi}$$

$$= \frac{A}{\pi}\left[\frac{-\cos(1+n)\pi + \cos(0)}{1+n} + \frac{-\cos(1-n)\pi + \cos(0)}{1-n}\right]$$

$$= \frac{A}{\pi}\left[\frac{-\cos(1+n)\pi + 1}{1+n} + \frac{-\cos(1-n)\pi + 1}{1-n}\right]$$

$$= \frac{A}{\pi}\left[\frac{-\cos(1+n)\pi + 1(1-n)}{1+n(1-n)} + \frac{-\cos(1-n)\pi + 1(1+n)}{1-n(1+n)}\right]$$

$1+n(1-n) = 1-n+n^2 \ldots$, $1-n^2$

$$= \frac{A}{\pi}\left[\frac{-\cos(1+n)\pi + 1(1-n) - \cos(1-n)\pi + 1(1+n)}{1-n^2}\right]$$

$$= \frac{A}{\pi}\left[\frac{\cos n\pi + 1 - n\cos n\pi - n + \cos n\pi + 1 + n\cos n\pi + n}{1-n^2}\right]$$

$$= \frac{A}{\pi}\left[\frac{2\cos n\pi + 2}{1-n^2}\right]$$

$$= -\frac{2A}{\pi}\left[\frac{\cos n\pi + 1}{n^2-1}\right]$$

When n is even, When n is odd,

$$a_n = \frac{-4A}{\pi(n^2-1)} \qquad a_n = 0$$

$$f(\omega t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n(\omega t) + \sum_{n=1}^{\infty} b_n \sin n(\omega t)$$

$$= \frac{1}{2}\frac{4A}{\pi} + \sum_{n=\text{even}} -\frac{4A}{\pi(n^2-1)}\cos n(\omega t)$$

$$f(\omega t) = \frac{2A}{\pi} + \sum_{n=\text{even}} -\frac{4A}{\pi(n^2-1)}\cos n(\omega t)$$ #

2. Calculate the fourier transform of the double pulse input function shown in figure 2.

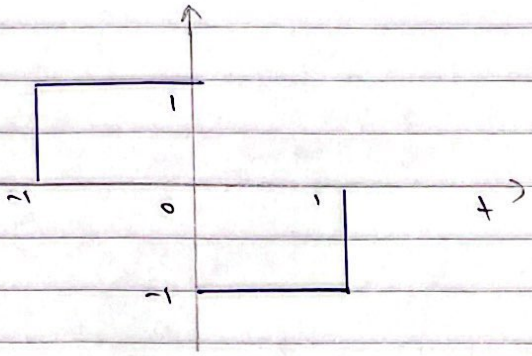

Figure 2

$$F(t) = u(t+1) - 2u(t) + u(t-1)$$

$$F(w) = \frac{1}{jw}e^{jw} - \frac{2}{jw} + \frac{e^{-jw}}{jw}$$

$$F(w) = \frac{1}{jw}\left[e^{jw} + e^{-jw}\right] - \frac{2}{jw}$$

$$F(w) = \frac{2}{jw}\left[\frac{e^{jw} + e^{-jw}}{2}\right] - \frac{2}{jw}$$

$$F(w) = \frac{2}{jw}\sin w - \frac{2}{jw}$$

$$F(w) = \frac{2}{jw}\left[\sin w - 1\right]$$ ǂ