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AM12 2

1. Derive the DFT of the sample data sequence $x(n) = \{1, 2, 1, 0\}$ and compute the responding amplitude and phase spectrum.

The N -point DFT of a finite duration sequence $x(n)$ is defined as

$$X[k] = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k=0, 1, 2, \dots, N-1$$

① for $k=0$

$$X[0] = \sum_{n=0}^3 x(n) e^{-j\pi(0)n/4}$$

$$= \sum_{n=0}^3 x(n) = 1 + 2 + 1 + 0 = 4$$

② for $k=1$

$$X[1] = \sum_{n=0}^3 x(n) e^{-j2\pi(1)n/4}$$

$$= \sum_{n=0}^3 x(n) e^{-j\pi n/2}$$

$$= 1 + 2e^{-j\pi/2} + 1e^{-j3\pi/2} + 0$$

$$= 1 + 2e^{-j\pi/2} + 1e^{j3\pi/2}$$

(2) for $k=2$

$$\begin{aligned}
 X[2] &= \sum_0^3 x(n) e^{-j2\pi(2)n/4} \\
 &= 1 + 2e^{-j2\pi/2} + 1e^{-j3\pi/2} \\
 &= 1 + 2e^{-j\pi} + 1e^{j\frac{3}{2}\pi}
 \end{aligned}$$

(3) for $k=3$

$$\begin{aligned}
 X[3] &= 1 + 2e^{-j3\pi/2} + 1e^{j3\pi/2} \\
 &= 1 + 2e^{j\frac{3\pi}{2}} + 1e^{-j2\pi}
 \end{aligned}$$

Amplitude spectrum

$$|X[k]| = \begin{cases} \sqrt{4+4} & \sqrt{1+2e^{-j\pi} + 1e^{j3\pi/2}} & \sqrt{1+2e^{-j\pi} + 1e^{j\frac{3}{2}\pi}} \\ \sqrt{1+2e^{j\frac{3\pi}{2}} + 1e^{-j2\pi}} & + 0 & \end{cases}$$

Phase spectrum

convert
rectangular form

$$\angle X[k] = \left\{ \tan^{-1}(0), \tan^{-1} \left(\frac{2e^{-j\pi/2} + 3e^{j3\pi/2}}{1} \right), \tan^{-1} \left(\frac{1e^{-j\pi}}{1} \right), \tan^{-1} \left(\frac{2e^{j\frac{3\pi}{2}} + 1e^{-j2\pi}}{1} \right) \right\}$$

$$= \left\{ 0, -\frac{\pi}{3}, -\frac{\pi}{6}, 0 \dots \right\}$$

✱